# DESCRIPTION OF AN IONOSPHERIC SPATIAL GRADIENT IRREGULARITY DETECTOR FOR WAAS*

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## BIOGRAPHIES

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## ABSTRACT

In the Wide Area Augmentation System (WAAS), one of the functions is the generation of the ionospheric delay estimates and an estimate of the post correction error bounds. However, ensuring with a high degree of confidence that the post-correction error bounds meet their integrity requirement has proven to be more difficult than anticipated. The difficulty stems mostly from the wide range of ionospheric conditions that the system may face over its lifetime. Given that the conditions that would challenge the system the most (severe geomagnetic storms) are relatively rare, system designers have moved toward an initial design constraining the availability of the service to non-severe ionospheric conditions. To that end, the design has been augmented with ionospheric irregularity detectors. Various types of ionospheric irregularity detectors have been examined. Among these are spatial gradient, temporal gradient, and Chi-square detectors. This paper discusses the initial design and the performance of a spatial gradient irregularity detector.

## INTRODUCTION

Satelite-Based Augmentation Systems (SBASs) include the U.S. Wide Area Augmentation System (WAAS), the European Geostationary Navigation Overlay System (EGNOS), and the Japanese MTSAT Satellite-based Augmentation System (MSAS). These systems generate correction vectors for the GPS satellite clock and ephemeris and for the ionospheric delays.

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They also provide post-correction residual error bounds. Corrections and residual error bounds are generated in SBAS Master Stations and broadcast to the users using geostationary orbiting communication satellites. The ionospheric corrections are referenced to ionospheric grid points (IGPs) on an imaginary grid 350 km above the earth's surface. SBAS integrity requirements are very stringent (i.e., potentially hazardous conditions must be bounded with a probability of 0.9999999 per approach, i.e., 150 seconds) and proving that a given design meets this requirement is very difficult considering the wide range of ionospheric conditions that might occur. Therefore, an initial WAAS goal is to use the ionospheric corrections only during periods of non-severe ionospheric conditions. Different types of ionospheric irregularity detectors have been proposed. Among these are spatial gradient, temporal gradient, and Chi-square detectors. This paper discusses the expected performance of a spatial gradient irregularity detector. The analysis is based on actual ionospheric data recorded by the WAAS network. The other types of detectors are analyzed in other papers.

## ANALYSIS

The sequential detector concept is illustrated in Figure 1 as described in References 1 and 2. In the sequential detector two hypotheses are set up corresponding to a quiet and disturbed ionospheric conditions. The hypothesis parameters are set by the false-alarm probability ($P_{fa}$) as well as the miss-detection probability ($P_m$). Data samples are accumulated until one of the thresholds is crossed. The main advantage of the sequential detector over the fixed sample size detector is that the sequential detector requires a smaller sample size (faster) to cross the thresholds for the same false alarm and miss detection probabilities.

Assume the following:

$x$ = the average spatial gradient (absolute value) around an IGP (m/100km) which has been found to have approximately a log-normal distribution with parameters $\mu$ and $\sigma$,
$P_m$ = miss detection probability, and
$P_{fa}$ = false alarm probability,

## SEQUENTIAL TEST:

The detector is testing two hypotheses:

$H_0 : f(x) = f(x; \mu_0, \sigma_0)$ corresponding to a quiet ionosphere,
$H_1 : f(x) = f(x; \mu_1, \sigma_1)$ corresponding to a disturbed ionosphere $(\mu_1 \geq \mu_0, \sigma_1 \geq \sigma_0)$,

where $f(x) = \frac{1}{\sqrt{2\pi}x\sigma} e^{-\frac{(\ell n(x)-\mu)^2}{2\sigma^2}}, x \geq 0$, is the probability density function (pdf) for a log-normal distribution (mean = $E(x) = e^{\mu + \frac{\sigma^2}{2}}$, variance = $V(x) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$, and the cumulative distribution function (CDF) = $\Phi\left(\frac{\ell n x - \mu}{\sigma}\right)$) [3].

The test it performs can be summarized as follows:

For epoch (sample) i=1, calculate the metric

$$z_n = \sum_{i=1}^{n=1} \left(\ell n(x_i) - b\right)^2 \text{, where } b = \frac{\frac{\mu_1}{\sigma_1^2} - \frac{\mu_0}{\sigma_0^2}}{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}},$$

if $z_n \leq \zeta_b$, accept $H_0$ (declare a quiet ionosphere),
if $z_n \geq \zeta_a$, accept $H_1$ (declare a disturbed ionosphere), and
if $\zeta_b < z_n < \zeta_a$, there is no enough information, take another sample and calculate

$z_n = \sum_{i=1}^{n} (\ell n(x_i) - b)^2$ . Repeat the test until one of the thresholds is crossed.

The thresholds are given by (see Appendix A):

$\zeta_a = h_a + ns$ , and
$\zeta_b = h_b + ns$

where

$$h_a = -\frac{2\ell nA}{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}} = -\frac{2\ell n\left(\frac{1-P_m}{P_{fa}}\right)}{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}}$$

$$h_b = -\frac{2\ell nB}{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}} = -\frac{2\ell n\left(\frac{P_m}{1-P_{fa}}\right)}{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}}$$

$$s = -\frac{2\ell n\left(\frac{\sigma_1}{\sigma_0}\right)}{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}} + b^2 - \frac{\frac{\mu_1^2}{\sigma_1^2} - \frac{\mu_0^2}{\sigma_0^2}}{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}} = slope$$

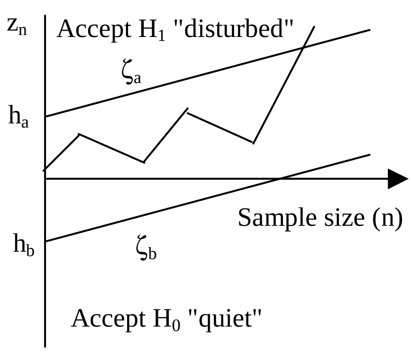


**Figure 1. An illustration of the sequential detector concept (two thresholds).**

**EXAMPLE:**

Figures 2 and 3 show histograms for the average spatial gradient in CONUS during quiet and disturbed ionospheric conditions respectively. The Regress+ [4] software was used to estimate the probability density functions. Bimodal normal distribution was the best fit to the data and the log-normal was the second best. We analyzed the log-normal distribution because the average spatial gradient is always greater than or equal to 0m/100km since we are only concerned with the magnitude of the absolute value, and also it was easier. The log-normal parameters are shown in Table 1.

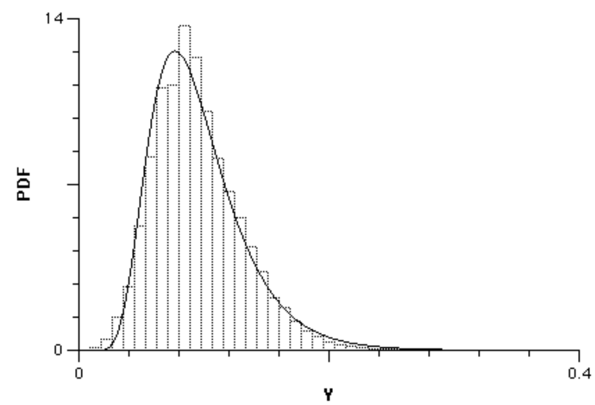


**Figure 2. Histogram for the average spatial gradient during a quiet ionosphere (log-normal fit parameters: $\mu_0$ = -2.41937, and $\sigma_0$ = 0.381707)(mean of x = E(x) = 0.09570 m/100km, standard deviation of x = 0.03790 m/100km).**

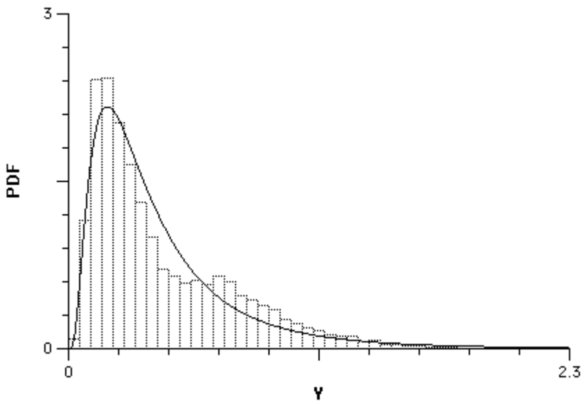


**Figure 3. Histogram for the average spatial gradient during a severe storm (log-normal fit parameters: $\mu_1$ = -1.16054 and $\sigma_1$ = 0.797590) (mean of x = E(x) = 0.43065 m/100km, standard deviation of x = 0.40609 m/100km).**

**Table 1. Log-normal distribution fit parameters for quiet and disturbed ionosphere.**

| | Quiet Ionosphere | Disturbed Ionosphere |
|---|---|---|
| Dates of data used to fit the probability density functions | 12/6/1999 | 4/6/2000 PM, and 4/7/2000 AM |
| Kp Index | See Figure 4 | See Figure 4 |
| $\mu$ | -2.41937 | -1.16054 |
| $\sigma$ | 0.381707 | 0.79759 |
| Mean of x (m/100km) | 0.0957 | 0.43065 |
| Standard deviation of x (m/100km) | 0.0379 | 0.40609 |

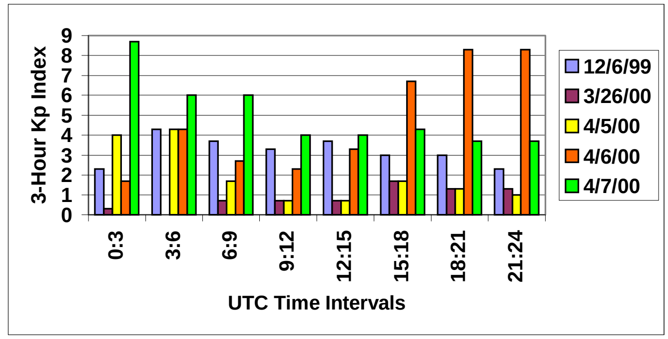


**Figure 4. The 3-hour Kp index for five days.**

Figure 5 shows the upper and lower thresholds as function of the sample size, n, for the same $P_{fa}$ ($1x10^{-3}$) and two different values of $P_m$ ($1x10^{-7}$ and $1x10^{-3}$). It is obvious from this figure that the lower threshold ($\zeta_b$) is more sensitive to $P_m$ than the upper threshold ($\zeta_a$). Figure 6 shows the normalized thresholds ($\zeta_b/n$) and ($\zeta_a/n$).

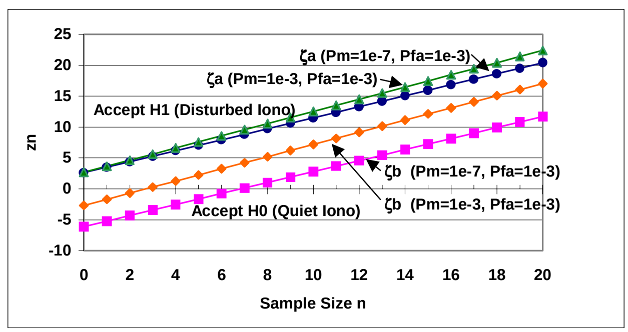


**Figure 5. Upper and lower thresholds for the sequential detector**

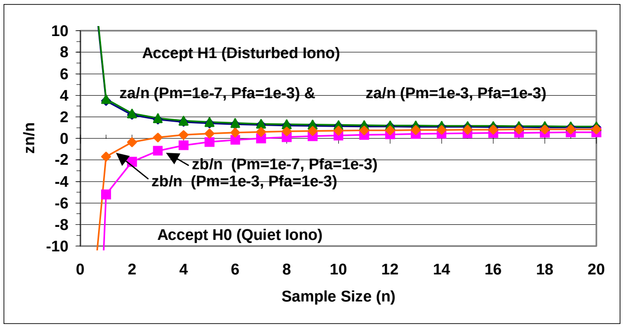


**Figure 6. Normalized upper and lower thresholds for the sequential detector.**

## SAMPLE RESULTS

This detector was tested using several days of recorded data that included both quiet and disturbed ionospheric conditions. Some limited results are presented in this paper simply to illustrate the concept of the detector. The false alarm and miss-detection probabilities are assumed to be $10^{-3}$ in these examples.

The top panel of Figure 7 shows the average spatial gradient around an ionospheric grid point (IGP) located near Atlantic City, NJ (40°N, 80°W). The average spatial gradient is calculated by taking the mean value of the gradients between pairs of ionospheric pierce points (IPPs) within a circular search area around the IGP with radius of 1600 km. Only pairs between IPPs that are at least 100 km apart were used for this computation. This restriction was imposed in order to minimize the effect of measurement noise. The data was obtained from the WAAS reference stations. The figure shows that the average spatial gradient remained very small (approximately equal 0.1m/100km) most of the day. The lower panel of Figure 7 shows the average sample number (ASN) that was needed to cross the lower threshold indicative of a quiet ionosphere. On average, it took 4-5 samples for the detector to declare the ionosphere around the IGP to be in the quiet state.

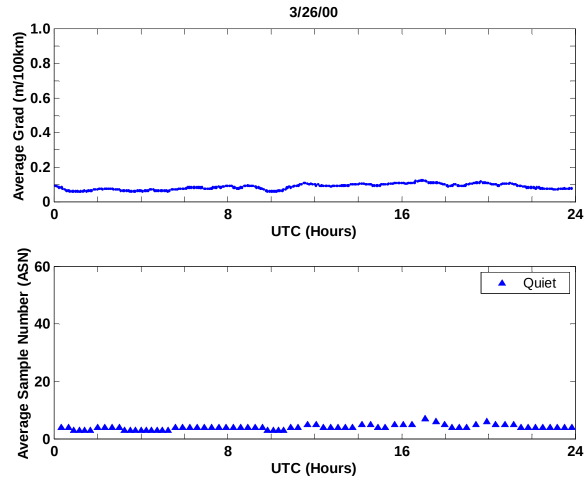


**Figure 7. Example of a Quiet Day (3/26/00) at IGP (40°N, 80°W) ($P_m$=1x10$^{-3}$ and $P_{fa}$ = 1x10$^{-3}$).**

Figure 8 shows the results for April 6, 2000. During the morning on that day, the ionosphere was quiet. The 3-hour kp index was ≤ 4 (see Figure 4), and the average spatial gradient was ≤ 0.2 m/100km at the IGP (40°N, 80°W) (see the top panel of Figure 8). At UTC 15:00-24:00, the Kp index starts to increase to 6.67 and then to 8.3, where it remains for the rest of the day. The average spatial gradient reached about 0.9 m/100km, which is about 9 times larger than during a typical quiet day (3/26/00). In the lower panel of Figure 8, the detector declares a quiet ionosphere between UTC 00:00 and 13:30. It took less than 5 samples (on average) for the detector to converge for every epoch in this period. When the average spatial gradient was large (after UTC 20:00), it took about one sample to declare the ionosphere in the disturbed state. Between 13:30 and 20:00 UTC, the detector took between 15 and 34 samples to do the same. During this time, the spatial gradient was about 0.2 m/100km (which is twice its value during quiet days). It seems that the value of the spatial gradient was in a transition between the quiet state and the disturbed state, which caused the detector to remain in the "not enough information" state longer. (To reduce this effect, it is possible to put maximum limit on the sample size, i.e., $n_{max}$ = 20. If the detector cannot make decision after $n_{max}$, then a disturbed ionosphere could be declared.)

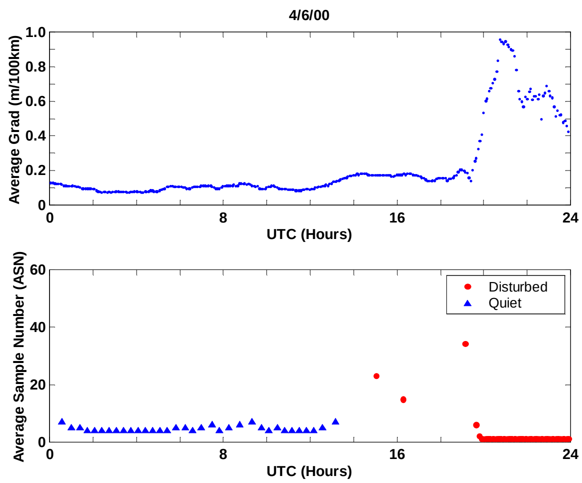


**Figure 8. Example of a disturbed day (4/6/00) at IGP (40°N, 80°W)($P_m$=1x10$^{-3}$ and $P_{fa}$ = 1x10$^{-3}$).**

Figure 9 shows the continuation of the storm which started on 4/6/00 PM. The average spatial gradient varies and the detecor responded to this variation between "disturbed", "quiet', and then "disturbed" again.

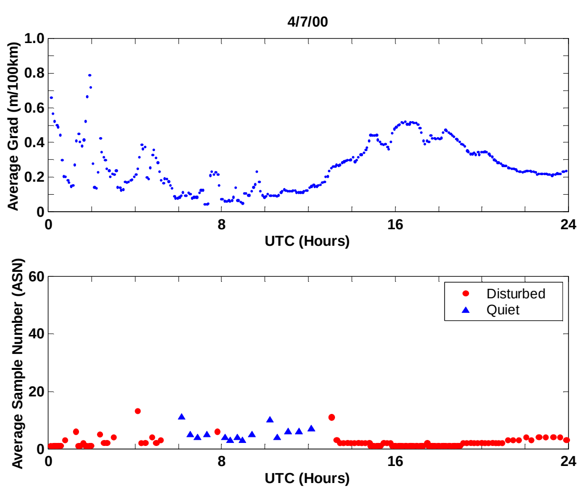


**Figure 9. Continuation of a disturbed day (4/7/00) at IGP (40°N, 80°W)($P_m$=1x10$^{-3}$ and $P_{fa}$ = 1x10$^{-3}$).**

## ACKNOWLEDGMENTS

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## LIST OF REFERENCES

1. Ghosh, B. K., *Sequential Tests of Statistical Hypotheses*, Addison-Wesley Publishing Company, Reading, MA, 1970.
2. Sage, A. P., and J. L. Melsa, *Estimation Theory With Applications to Communications and Control*, McGraw-Hill Book Company, New York, NY, 1971.
3. Kotz, S., and N. L. Johnson, Encyclopedia of Statistical Sciences, Vol. 5., p. 134, New York, N.Y., 1985.
4. Regress+ software and its documentation for the Macintosh computers: http://www.geocities.com/~mikemclaughlin/software/Regress_plus.html,http://www.geocities.com/~mikemclaughlin/math_stat/Dists/Compendium.html.

## APPENDIX A

## SEQUENTIAL TEST:

We are testing two hypotheses:

$H_0 : f(x) = f(x;\mu_0,\sigma_0)$ for a quiet ionosphere

$H_1 : f(x) = f(x;\mu_1,\sigma_1)$ for a disturbed ionosphere $(\mu_1 \geq \mu_0, \sigma_1 \geq \sigma_0)$

where $f(x) = \frac{1}{\sqrt{2\pi}x\sigma} e^{-\frac{(\ell n(x)-\mu)^2}{2\sigma^2}}, x \geq 0$, is the probability density function (pdf) for a log-normal distribution (mean = $E(x) = e^{\mu+\frac{\sigma^2}{2}}$, variance = $V(x) = e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$, CDF = $\Phi\left(\frac{\ell n(x)-\mu}{\sigma}\right)$)[3].

The likelihood ratio for n samples is given by:

$$\lambda_n = \prod_{i=1}^{n} \frac{f(x_i;\mu_1,\sigma_1)}{f(x_i;\mu_0,\sigma_0)} =$$

$$\left(\frac{\sigma_0}{\sigma_1}\right)^n \exp\left[-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{(\ell n(x_i)-\mu_1)^2}{\sigma_1^2}-\frac{(\ell n(x_i)-\mu_0)^2}{\sigma_0^2}\right)\right]$$

With some mathematical manipulations, the logarithm of the likelihood ratio becomes:

$$\ell n(\lambda_n) = n\left\{\ell n\left(\frac{\sigma_0}{\sigma_1}\right)+\frac{1}{2}\left(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}\right)\left[b^2-\left(\frac{\frac{\mu_1^2}{\sigma_1^2}-\frac{\mu_0^2}{\sigma_0^2}}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}}\right)\right]\right\}$$

$$-\frac{1}{2}\left(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}\right)\sum_{i=1}^{n}(\ell n(x_i)-b)^2$$

where $b = \dfrac{\frac{\mu_1}{\sigma_1^2}-\frac{\mu_0}{\sigma_0^2}}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}}$.

Define A and B such that:

$A = \dfrac{1-P_m}{P_{fa}}$ and $B = \dfrac{P_m}{1-P_{fa}}$. Therefore the sequential test is described by:

If $\lambda_n \le B$, accept $H_0$ (i.e., a quiet ionosphere),
If $\lambda_n \ge A$, accept $H_1$ (i.e., a disturbed ionosphere), and
If $B < \lambda_n < A$, there is no enough information, take another sample and repeat the test until one of the threshold is crossed.

A similar test can be done on the logarithm of the likelihood ratio. Therefore:

If $\ell n(\lambda_n) \le \ell n(B)$, accept $H_0$ (i.e., a quiet ionosphere),
If $\ell n(\lambda_n) \ge \ell n(A)$, accept $H_1$ (i.e., a disturbed ionosphere), and
If $\ell n(B) < \ell n(\lambda_n) < \ell n(A)$, there is no enough information, take another sample and repeat the test until one of the threshold is crossed.

Therefore the sequential test becomes:

For epoch (sample) i=1, calculate the metric

$$z_n = \sum_{i=1}^{n=1}(\ell n(x_i)-b)^2$$

If $z_n \le \zeta_b$, accept $H_0$ (declare a quiet ionosphere),
If $z_n \ge \zeta_a$, accept $H_1$ (declare a disturbed ionosphere),
If $\zeta_b < z_n < \zeta_a$, there is no enough information, take another sample and calculate $z_n = \sum_{i=1}^{n}(\ell n(x_i)-b)^2$. Repeat the test until one of the thresholds is crossed.

The thresholds are given by:

$\zeta_a = h_a + ns$, and
$\zeta_b = h_b + ns$

where

$$h_a = -\frac{2\ell nA}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}} = -\frac{2\ell n\left(\frac{1-P_m}{P_{fa}}\right)}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}}$$

$$h_b = -\frac{2\ell nB}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}} = -\frac{2\ell n\left(\frac{P_m}{1-P_{fa}}\right)}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}}$$

$$s = -\frac{2\ell n\left(\frac{\sigma_1}{\sigma_0}\right)}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}} + b^2 - \frac{\frac{\mu_1^2}{\sigma_1^2}-\frac{\mu_0^2}{\sigma_0^2}}{\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}} = slope$$